Three-Dimensional Flow Arising in the Long Porous Slider: An Analytic Solution

Yasir Khan[a], Qingbiao Wu[a], Naeem Faraz[b], Ahmet Yildirim[c], and Syed Tauseef Mohyud-Din[d]

[a] Department of Mathematics, Zhejiang University, Hangzhou 310027, China
[b] Modern Textile Institute, Donghua University, 1882 Yan'an Xilu Road, Shanghai 200051, China
[c] Ege University, Science Faculty, Department of Mathematics, 35100 Bornova Izmir, Turkey
[d] HITEC University Taxila Cantt, Pakistan

Reprint requests to Y. K.; E-mail: yasirmath@yahoo.com

Z. Naturforsch. **66a,** 507 – 511 (2011) / DOI: 10.5560/ZNA.2011-0008
Received January 4, 2011 / revised April 13, 2011

In this work, the long porous slider problem where the fluid is injected through the porous bottom is studied. The similarity transformations reduce the equation of motion to a set of nonlinear ordinary differential equations which are solved using the Adomian decomposition method (ADM). The influence of the Reynolds number on the dimensionless velocity field has been discussed graphically. Finally, the validity of results is verified by comparing with the numerical method and existing numerical results. A very good agreement was found between approximate and numerical solution, which proves that ADM is very efficient and accurate.

Key words: Adomian Decomposition Method (ADM); Lubrication; Porous Slider; Reynolds Number.

1. Introduction

Porous materials are widely used in chromatography, chemical reactions, heat transfer, and analytical chemistry filtering analyte due to their large surface area and small pore sizes. In addition, fluid flow through porous channels can cause less dispersion to an analyte than fluid flow through an open channel of the same dimensions. Much work has been done in order to understand the effects of fluid removal or injection through channel walls on the flow of Newtonian and non-Newtonian fluids. In view of these applications, Berman [1] made an initial effort in this direction. His investigations provided a technique for solving the classical viscous flow equations. The flow problem between porous plates has been studied extensively in various aspects, for example non-Newtonian fluids, magnetohydrodynamic (MHD) flows, heat transfer and mass transfer analysis. The literature on the topic is quite extensive and hence can not be described here in detail. However, some most recent works of eminent researchers regarding the flow between porous plates may be mentioned in the studies [2 – 14]. To gain insight into the real situation an attempt is made for the analysis of the three-dimensional problem involving Reynolds number [15 – 20]. Shalak and Wang [3] carried out a numerical analysis of the problem for moderately large Reynolds numbers. The Reynolds number can be defined for a number of different situations where a fluid is in relative motion to a surface (the definition of the Reynolds number is to be confused with the Reynolds equations or lubrication equation).

In order to overcome the restrictions of perturbation techniques, some non-perturbation methods were developed such as the Laplace decomposition method [21 – 23], the bookkeeping artificial parameter method [24], the energy balance method [25], the parameter-expansion method [26], the variational iteration method [27, 28] and so on. One of those non-perturbation methods is the Adomian decomposition method (ADM) proposed by Adomian [29]. A reliable modification of the Adomian decomposition algorithm has been done by Wazwaz [30]. The current paper considers a three-dimensional problem using the Adomian decomposition method. The Adomian decomposition method [31, 32] is quantitative rather than qualitative analytic, requiring neither linearization nor perturbation and continuous with no resort to discretization. The method has been used to derive analytical solutions for nonlinear ordinary differential equations [33 – 35] as well as partial differential equations [36 – 44]. A modified version of the method was used to derive the analytic solutions for partial and ordinary differential equations [45, 46]. To the best of our knowledge no attempt has been made to exploit this method to solve the long porous slider problem. Also our aim in this article is to compare the results with solutions to the existing ones [3].

2. Formulation of the Problem

Consider a long porous slider with dimensions L_1 and L_2 (Fig. 1 a and b). A fluid is injected through the porous bottom of the slider with velocity W^* such that a small gap of width d is created. The slider moves laterally with velocity $-U$ and longitudinally with velocity $-V$ in the x and y-directions, respectively (Fig. 1 a). We shall assume $L_2 \geq L_1 \geq d$ such that end effects can be neglected. In a reference frame travelling with the slider let u, v, and w be the velocity components of the fluid in the x, y, and z-direction, respectively. The basic governing equations of the problem given by Shalak and Wang [3] are:

$$\nabla \cdot q = 0, \tag{1}$$

$$(q \cdot \nabla) q = -\frac{\nabla p}{\rho} + \gamma \nabla^2 q, \tag{2}$$

where $q = (u, v, w)$, p is the pressure, ρ is the density of fluid, and γ is the kinematic viscosity. The boundary conditions for (1) and (2) are [3]:

$$\begin{aligned} &u = U, \ v = V, \ w = 0 \quad && \text{at } z = 0, \\ &w = -W^*, \ u = v = 0 \quad && \text{at } z = d, \end{aligned} \tag{3}$$

where U, V are the velocities of the slider in lateral and longitudinal directions, and W^* is the velocity of the fluid injected through the porous bottom of the slider. Using the similarity transformation [3]

$$\begin{aligned} &u = U f(\eta) + \frac{W^* x}{d} h'(\eta), \ v = V g(\eta), \\ &w = -W^* h(\eta), \end{aligned} \tag{4}$$

with $\eta = z/d$, the Navier–Stokes equations reduce to [3]

$$h'h'' - hh''' = \frac{h''''}{R}, \tag{5}$$

$$fh' - hf' = \frac{f''}{R}, \tag{6}$$

$$-hg' = \frac{g''}{R}, \tag{7}$$

$$\begin{aligned} &h(0) = h'(0) = 0, \ g(0) = f(0) = 1, \\ &h(1) = 1, \ h'(1) = g(1) = f(1) = 0, \end{aligned} \tag{8}$$

where $R = W^* d/\gamma$ is the Reynolds number. These equations differ completely from the circular case [2]. Following the standard procedure of the Adomian decomposition method defined in [29, 47], we can

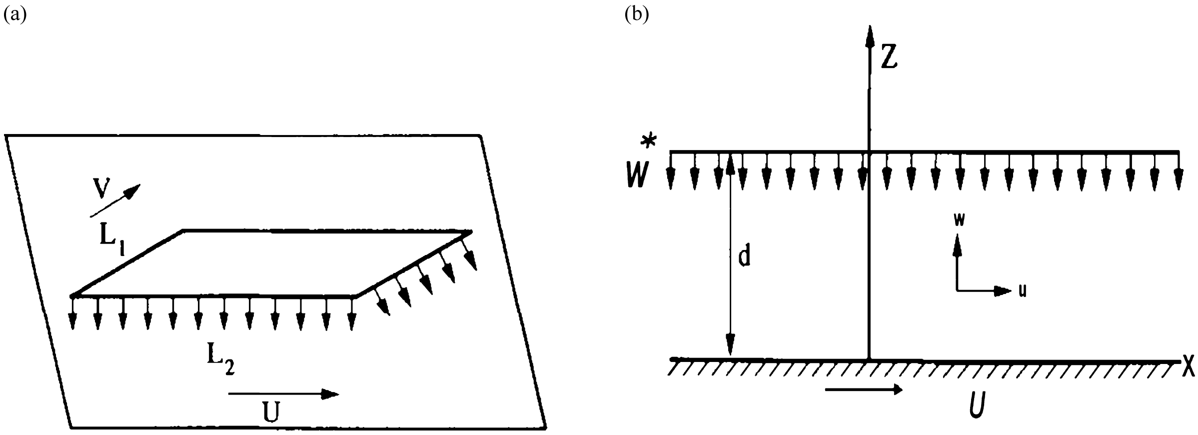

Fig. 1. (a) Moving long porous slider; (b) coordinate system.

write (5) to (7) as follows:

$$h_{n+1}(\eta) = \int_0^\eta \int_0^\eta \int_0^\eta \int_0^\eta R\left(\sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} B_n\right) \mathrm{d}\eta\mathrm{d}\eta\mathrm{d}\eta\mathrm{d}\eta, \tag{9}$$

$$f_{n+1}(\eta) = \int_0^\eta \int_0^\eta R\left(\sum_{n=0}^{\infty} C_n - \sum_{n=0}^{\infty} D_n\right) \mathrm{d}\eta\mathrm{d}\eta, \tag{10}$$

$$g_{n+1}(\eta) = -\int_0^\eta \int_0^\eta R\left(\sum_{n=0}^{\infty} E_n\right) \mathrm{d}\eta\mathrm{d}\eta. \tag{11}$$

A corresponding initial guess is given below by using the boundary conditions defined in (8):

$$h_0 = 3\eta^2 - 2\eta^3, \quad f_0 = g_0 = 1 - \eta^2. \tag{12}$$

3. Results and Discussion

In order to solve (9)–(11) subject to the initial guess (12) analytically, we use the Adomian decomposition method as described in the books by G. Adomian [29] and A. M. Wazwaz [47]. The method has the following main steps:

1. Splitting the given equation into linear and nonlinear parts.
2. Inverting the highest-order derivative operator contained in the linear operator on both sides.
3. Identifying the initial and/or boundary conditions and the terms involving the independent variables alone as initial approximation.
4. Decomposing the unknown function into a series whose components are to be determined.
5. Decomposing the nonlinear function in terms of special polynomials called Adomian's polynomials, and finding the successive terms of the series solution by recurrent relation using these Adomian polynomials.

The graphical behaviour of h, f, and g for different values of the Reynolds number are presented graphically for a 10th-order approximation calculated by using Mathematica. For the validation of the numerical solution used in this study, the results are compared with those of Shalak and Wang [3]. Using the inverse method, Wang [3] obtained results for the porous flat slider. The comparison is found to be very good.

The effects of the Reynolds number R on the velocity components are shown in Figures 2–4. It is seen

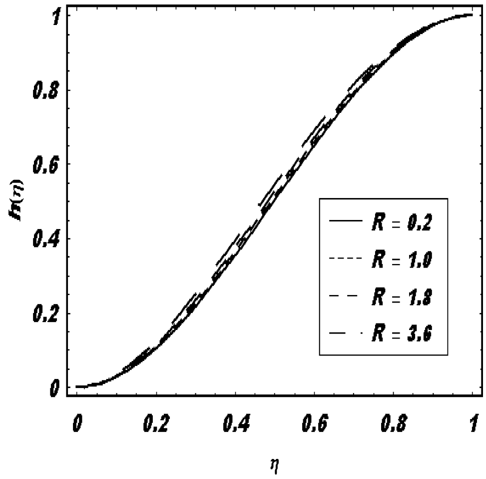

Fig. 2. Effects of R on h.

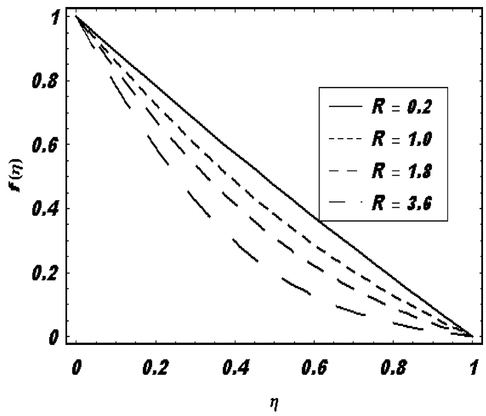

Fig. 3. Effects of R on f.

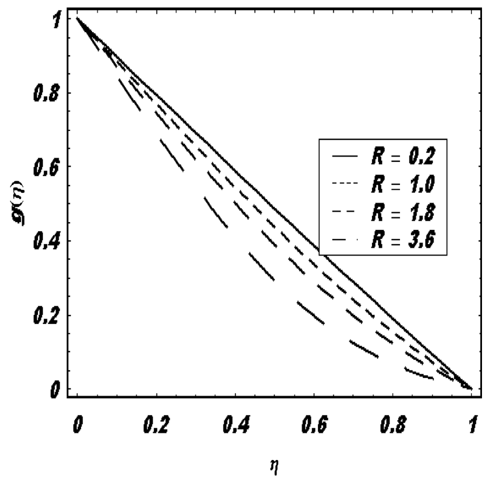

Fig. 4. Effects of R on g.

Table 1. Comparison between the ADM solution and the numerical solution for different values of the Reynolds number R.

R	ADM $h''(0)$	Numerical finite difference $h''(0)$	ADM $h'''(0)$	Numerical finite difference $h'''(0)$	ADM $f'(0)$	Numerical finite difference $f'(0)$	ADM $g'(0)$	Numerical finite difference $g'(0)$
0.2	6.09133	6.091327	−12.465	−12.4649925	−1.0880	−1.067376	−1.0301	−1.030147
1	6.45426	6.454247	−14.365	−14.365764	−1.4059	−1.340171	−1.1531	−1.153103
5	8.17386	8.173776	−24.586	−24.583043	−2.4417	−2.652627	−1.7666	−1.766410
13.8	11.26234	11.261533	−48.9043	−48.479130	−4.0229	−4.741560	−2.8074	−2.806604
51.6	19.449	19.4483819	−149.67	−149.666504	−7.553	−7.6789	−5.301	−5.30102

that h increases with the increasing values of R (Fig. 2). The variation of R on f is illustrated in Figure 3. This figure shows that with increasing values of R, f is decreasing. The effect of R on velocity field g is shown in Figure 4. Here, the velocity profile shows the same behaviour as compared to f.

Table 1 clearly reveals that the present solution method, namely ADM, shows excellent agreement with the existing solutions in literature [3] and numerical method solutions. This analysis shows that ADM suits for the long porous slider problem.

4. Conclusion

The case of three-dimensional lubrication of a long porous slider is discussed. The nonlinear system is solved analytically using the Adomian decomposition method (ADM). The effects of the Reynolds number is discussed through graphs. The case of lubrication of long porous slider via Adomian decomposition method has never been reported and the following observations have been made:

- h is an increasing function of the Reynolds number,
- f and g are decreasing functions of the Reynolds number,
- the effect of the Reynolds number is more prominent on h and g as on f.

Acknowledgement

The authors are grateful to the reviewers for their comments and useful suggestions. This work was supported by the National Natural Science Foundation of China (Grant No. 10871178) and the Foundation of Science and Technology Department of Zhejiang Province (Grant No. 2008C01048-3).

[1] A. S. Berman, J. Appl. Phys. **24**, 1232 (1953).
[2] C. Y. Wang, J. Appl. Mech. Trans. ASME **41**, 343 (1974).
[3] F. M. Shalak and C. Y. Wang, J. Appl. Mech. Trans. ASME **42**, 893 (1975).
[4] L. T. Watson, T. Y. Li, and C. Y. Wang, J. Appl. Mech. Trans. ASME **45**, 435 (1978).
[5] F. M. Shalak and C. Y. Wang, SIAM J. Appl. Math. **34**, 535 (1978).
[6] C. Y. Wang, J. Lubrication Technol. Trans. ASME **100**, 444 (1978).
[7] P. D. Ariel, Int. J. Numer. Meth. Fluids **17**, 605 (1993).
[8] J. H. He, J. Comput. Math. Appl. Mech. Eng. **167**, 57 (1998).
[9] G. Shantha and B. Shanker, Int. J. Numer. Meth. Heat Fluid Flow **20**, 250 (2010).
[10] Z. Z. Ganji and D. D. Ganji, Transp. Porous Med. **81**, 527 (2010).
[11] B. Raftari and A. Yıldırım, Comput. Math. Appl. **59**, 3328 (2010).
[12] D. Srinivasacharya, J. V. R. Murthy, and D. Venugopalam, Int. J. Heat Mass Transf. **39**, 1557 (2001).
[13] A. Postelnicu, Heat Mass Transf. **43**, 595 (2007).
[14] R. Cortell, Chem. Eng. Process. **46**, 721 (2007).
[15] D. B. DeGraaff, D. R. Webster, and J. K. Eaton, Exp. Therm. Fluid Sci. **18**, 341 (1999).
[16] M. Gad-el-Hak and P. R. Bandyopadhyay, Appl. Mech. Rev. **47**, 307 (1994).
[17] S. Mochizuki and F. T. M. Nieuwstadt, Exp. Fluids **21**, 218 (1996).
[18] J. Kim, P. Moin, and R. D. Moser, J. Fluid Mech. **177**, 133 (1987).
[19] G. Zhu and C. Wu, Commun. Nonlin. Sci. Numer. Simul. **2**, 165 (1997).
[20] Y. N. Huang, Commun. Nonlin. Sci. Numer. Simul. **9**, 543 (2004).

[21] Y. Khan, Int. J. Nonlin. Sci. Numer. Simul. **10**, 1373 (2009).
[22] Y. Khan and F. Austin, Z. Naturforsch. **65a**, 849 (2010).
[23] A. M. Wazwaz, Appl. Math. Comput. **216**, 1304 (2010).
[24] J. H. He, Int. J. Nonlin. Sci. Numer. Simul. **2**, 195 (2001).
[25] S. S. Ganji, D. D. Ganji, Z. Z. Ganji, and S. Karimpour, Acta Appl. Math. **106**, 79 (2009).
[26] F. O. Zengin, M. O. Kaya, and S. A. Demirbag, Int. J. Nonlin. Sci. Numer. Simul. **9**, 267 (2008).
[27] J. H. He, G. C. Wu, and F. Austin, Nonlin. Sci. Lett. A **1**, 1 (2010).
[28] N. Herişanu and V. Marinca, Nonlin. Sci. Lett. A **1**, 183 (2010).
[29] G. Adomian, Solving Frontier Problem of Physics: The Decomposition Method, Kluwer Acadamic Publisher, Dordrecht 1994.
[30] A. M. Wazwaz, Appl. Math. Comput. **102**, 77 (1999).
[31] G. Adomian, J. Math. Anal. Appl. **135**, 501 (1988).
[32] G. Adomian, Appl. Math. Lett. **11**, 121 (1998).
[33] M. M. Hosseini and M. Jafari, Commun. Nonlin. Sci. Numer. Simul. **14**, 1952 (2009).
[34] M. M. Hosseini, Appl. Math. Comput. **181**, 1737 (2006).
[35] M. M. Hosseini, J. Comput. Appl. Math. **197**, 495 (2006).
[36] M. J. Pujol and P. Grimalt, J. Numer. Meth. Heat Fluid Flow **13**, 473 (2003).
[37] H. Jafari and V. Daftardar-Gejji, Appl. Math. Comput. **180**, 700 (2006).
[38] M. Dehghan, Appl. Math. Comput. **157**, 549 (2004).
[39] M. Safari, D. D. Ganji, and M. Moslemi, Comput. Math. Appl. **58**, 2091 (2009).
[40] M. Tatari, M. Dehghan, and M. Razzaghi, Math. Comput. Modell. **45**, 639 (2007).
[41] M. A. Abdou, J. Quant. Spectros. Rad. Transf. **95**, 407 (2005).
[42] N. H. Sweilam and M. M. Khader, Appl. Math. Comput. **217**, 495 (2010).
[43] A. Sadighi and D. D. Ganji, Phys. Lett. A **367**, 83 (2007).
[44] E. Hetmaniok, D. Słota, R. Witua, and A. Zielonka, Comput. Math. Appl. **61**, 1931 (2011).
[45] A. M. Wazwaz and S. M. El-Sayed, Appl. Math. Comput. **122**, 393 (2001).
[46] A. M. Wazwaz, Appl. Math. Comput. **118**, 123 (2001).
[47] A. M. Wazwaz, Partial Differential Equations and Solitary Waves Theory, Springer, Dordrecht, Heidelberg, London, New York 2009.